Name: ______________________________ Period: ___________ Date: ________________

Operations with Radical Expressions Guide Notes

ANSWERS

Adding and Subtracting Radical Expressions

Only radicals with the same index and same radicand (like radicals) can be combined by addition or subtraction.

If the radicals are not in simplified form, then they must be simplified before you can determine whether they can be combined.

Note: When adding or subtracting radicals, the index and radicand do not change.

Sample Problem 1: Perform the indicated operations and simplify as completely as possible. Assume that all variables represent positive real numbers.

a.
$$2\sqrt{5}+3\sqrt{5}-\sqrt{5}=$$
$$=(2+3-1)\sqrt{5}=$$
$$=4\sqrt{5}$$

b.
$$\sqrt{125}-3\sqrt{80}+\sqrt{45}=$$
$$=\sqrt{25*5}-3\sqrt{16*5}+\sqrt{9*5}=$$
$$=5\sqrt{5}-12\sqrt{5}+3\sqrt{5}=$$
$$=-4\sqrt{5}$$

c.
$$4\sqrt[3]{x}+12\sqrt[3]{x}-5\sqrt[3]{x}=$$
$$=(4+12-5)\sqrt[3]{x}=$$
$$=11\sqrt[3]{x}$$

d.
$$3\sqrt{ab^2}-4b\sqrt{a}+\sqrt{4ab^2}=$$
$$=3b\sqrt{a}-4b\sqrt{a}+2b\sqrt{a}=$$
$$=(3b-4b+2b)\sqrt{a}=$$
$$=b\sqrt{a}$$

Multiplication of Radical Expressions

To multiply radical expressions, just multiply using the same rules as multiplying polynomials, (Distributive Property, FOIL and Exponent Rules) except NEVER multiply values outside the radical times values inside the radical.

Sample Problem 2: Perform the indicated operations and simplify as completely as possible. Assume that all variables represent positive real numbers.

a.
$$2\sqrt{3}(\sqrt{7}-6\sqrt{5})=$$
$$=2\sqrt{3}*\sqrt{7}-2\sqrt{3}*6\sqrt{5}=$$
$$=2\sqrt{21}-12\sqrt{15}$$

b.
$$(\sqrt{3}-3\sqrt{5})(2\sqrt{3}+\sqrt{5})=$$
$$=\sqrt{3}*2\sqrt{3}+\sqrt{3}*\sqrt{5}-3\sqrt{5}*2\sqrt{3}-3\sqrt{5}*\sqrt{5}$$
$$=6-\sqrt{15}+6\sqrt{15}-15=$$
$$=-9+5\sqrt{15}$$

c.
$$2\sqrt[3]{x}(12\sqrt[3]{x^2}-4x\sqrt[3]{x^5})=$$
$$=24\sqrt[3]{x^3}-8x\sqrt[3]{x*x^5}=$$
$$=24x-8x\sqrt[3]{x^6}=$$
$$=24x-8x^2$$

d.
$$(2x+2\sqrt{x})(\sqrt{x}-\sqrt{5x})=$$
$$=2x*\sqrt{x}-2x*\sqrt{5x}+2\sqrt{x}*\sqrt{x}-2\sqrt{x}*\sqrt{5x}=$$
$$=2x\sqrt{x}-2x\sqrt{5x}+2x-2\sqrt{x*5x}=$$
$$=2x\sqrt{x}-2x\sqrt{5x}+2x-2x\sqrt{5}$$

e.
$$(2\sqrt{3}-\sqrt{6})^2=$$

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$$= (2\sqrt{3} - \sqrt{6}) * (2\sqrt{3} - \sqrt{6}) =$$
$$= 2\sqrt{3} * 2\sqrt{3} - 2\sqrt{3} * \sqrt{6} - \sqrt{6} * 2\sqrt{3} + \sqrt{6} * \sqrt{6} =$$
$$= 4\sqrt{3^2} - 4\sqrt{18} + \sqrt{6^2} =$$
$$= 12 - 12\sqrt{2} + 6 =$$
$$= 18 - 12\sqrt{2}$$

When multiplying radicals with different indexes, change to rational exponents first, find a common denominator in order to add the exponents, then rewrite in radical notation.

Sample Problem 3: Perform the indicated operations and simplify as completely as possible. Assume that all variables represent positive real numbers.

a. $$\sqrt{5} * \sqrt[3]{25} = 5^{\frac{1}{2}} * 25^{\frac{1}{3}} = 5^{\frac{1*3}{2*3}} * 25^{\frac{1*2}{3*2}} = 5^{\frac{3}{6}} * 25^{\frac{2}{6}} = \sqrt[6]{5^3} * \sqrt[6]{25^2} = \sqrt[6]{5^3 * 5^4} = \sqrt[6]{5^6 * 5} = 5\sqrt[6]{5}$$

b. $$\sqrt[3]{x} * \sqrt[5]{x} = x^{\frac{1}{3}} * x^{\frac{1}{5}} = x^{\frac{1*5}{3*5}} * x^{\frac{1*3}{5*3}} = x^{\frac{5}{15}} * x^{\frac{3}{15}} = \sqrt[15]{x^5} * \sqrt[15]{x^3} = \sqrt[15]{x^5 * x^3} = \sqrt[15]{x^8}$$

Dividing Radical Expressions (Rationalizing the Denominator)

Sample Problem 4: Perform the indicated operations and simplify as completely as possible. Assume that all variables represent positive real numbers.

a. $$\frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} * \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

b. $$\frac{3\sqrt[3]{x}}{\sqrt[3]{y}} = \frac{3\sqrt[3]{x}}{\sqrt[3]{y}} * \frac{\sqrt[3]{y^2}}{\sqrt[3]{y^2}} = \frac{3\sqrt[3]{xy^2}}{\sqrt[3]{y^3}} = \frac{3\sqrt[3]{xy^2}}{y}$$

c. $$\frac{\sqrt{x}}{1 + \sqrt{x}} = \frac{\sqrt{x}}{1 + \sqrt{x}} * \frac{1 - \sqrt{x}}{1 - \sqrt{x}} = \frac{\sqrt{x}(1 - \sqrt{x})}{(1 + \sqrt{x})(1 - \sqrt{x})} = \frac{\sqrt{x} - \sqrt{x^2}}{1 - \sqrt{x} + \sqrt{x} - \sqrt{x^2}} = \frac{\sqrt{x} - x}{1 - x}$$

